Exercise 2.1.3

In the next three exercises, interpret $\dot{x} = \sin x$ as a flow on the x-axis.

(Acceleration)

a) Find the flow's acceleration $\ddot{x}$ as a function of x.

b) Find the points where the flow has maximum positive acceleration.

Solution

Differentiate both sides with respect to t.

$$\begin{aligned}
\frac{d}{dt}(\dot{x}) &= \frac{d}{dt}(\sin x) \\
\ddot{x} &= (\cos x) \cdot \frac{d}{dt}(x) \\
&= (\cos x) \cdot \dot{x} \\
&= (\cos x)(\sin x) \\
&= \frac{1}{2} \sin 2x
\end{aligned}$$

The greatest acceleration to the right occurs where $\ddot{x}$ is maximum (and positive), that is, where $\sin 2x = 1$:

$$\begin{aligned}
2x &= \frac{\pi}{2} + 2n\pi, \quad n = 0, \pm 1, \pm 2, \ldots \\
x &= \frac{\pi}{4} + n\pi.
\end{aligned}$$